Curriculum Vitæ et Studiorum
Carlo Petronio

May 26, 2026

Contents

1 Titles **2**

2 Positions **2**

3 Teaching **3**

4 Dissertations supervised **8**

5 Invited talks and advanced courses **11**

6 Conferences attended **16**

7 Administrative and organizational activities **16**

7.1 Main services 16

7.2 Participation to selection and evaluation committees 17

7.3 Organization of scientific events 19

7.4 Direction of research projects 20

7.5 Other administrative offices 21

8 General information **23**

9 Scientific activity: completed works **24**

10 Scientific activity: work in progress and projects **38**

1 Titles

March 31, 1995. PhD degree *cum laude* from the Scuola Normale Superiore di Pisa. Dissertation entitled *Standard spines and* 3*-manifolds* supervised by Riccardo Benedetti. Evaluation committee: Riccardo Benedetti, Michel Boileau, Corrado de Concini, Maria Dedò, Margherita Galbiati, Hugh Morton, Edoardo Vesentini.

April 23, 1994. Francesco Severi prize.

July 13, 1989. First degree in Mathematics *cum laude* from the University of Pisa; dissertation entitled *Variations on a theme of Thullen: non-homogeneous bounded domains in Banach spaces*, supervised by Edoardo Vesentini.

July 13, 1989. Diploma from the Scuola Normale Superiore di Pisa.

2 Positions

2025/26 – today. Full professor of geometry at the Department of Mathematical, Physical and Computer Sciences, University of Parma.

2012/13 – 2024/25. Full professor of geometry at the Department of Mathematics, University of Pisa.

1992 – today. Secretary of the Editorial Board, Annali della Classe di Scienze, Scuola Normale Superiore di Pisa.

July 2017 – June 2025. Member of the scientific council of GNSAGA.

2002/03 – 2011/12. Full professor of geometry at the Faculty of Engineering, University of Pisa.

February – March 2007. Invited professor at the University of Toulouse.

May 2003. Invited professor at the University of Paris VII.

1999/00 – 2001/02. Associate professor of geometry at the Faculty of Engineering, University of Pisa.

1993/94 – 1998/99. Research assistant at the Faculty of Science, University of Pisa.

1992/93. Civil service (as a conscientious objector to the military service).

1991/92. Visiting graduate student at the Mathematics Institute of the University of Warwick (UK).

1989/90 – 1991/92. Graduate student of Mathematics at the Scuola Normale Superiore di Pisa.

1985/86 – 1988/89. Student of Mathematics, Scuola Normale Superiore di Pisa and University of Pisa.

3 Teaching

2026/2027.

- "Matematica II," first year, degree in Scienza dei Materiali, University of Parma (56 hours).
- "Matematica con Elementi di Statistica," first year, degree in Pharmaceutical and Chemistry and Technology, University of Parma (64 hours).

2025/2026.

- "Matematica II," first year, degree in Scienza dei Materiali, University of Parma (56 hours).
- "Matematica con Elementi di Statistica," first year, degree in Pharmaceutical and Chemistry and Technology, University of Parma (64 hours).
- "Matematica con Elementi di Statistica," first year, degrees in Pharmacy, University of Parma (56 hours — contained in the previous one).
- "Algebra and Geometry," first year, degree in Computer Science, University of Parma (48 hours).

2024/2025.

- "Istituzioni di Matematica I," first year, degree in Chimica per l'Industria e l'Ambiente, University of Pisa (60 hours).
- "Geometry," first year, degree in Civil Engineering, University of Pisa (40 hours).

2023/2024.

- "Istituzioni di Matematica I," first year, degree in Chimica per l'Industria e l'Ambiente, University of Pisa (80 hours).
- "Geometry," first year, degree in Civil Engineering, University of Pisa (40 hours).

2022/2023.

- "Istituzioni di Matematica I," first year, degree in Chimica per l'Industria e l'Ambiente, University of Pisa (60 hours).
- "Geometry," first year, degree in Civil Engineering, University of Pisa (60 hours).

2021/2022.

- "Linear Algebra," first year, degree in Biomedical Engineering, University of Pisa (20 hours).
- "Geometry," first year, degree in Civil Engineering, University of Pisa (40 hours).

2020/2021.

- "Linear Algebra," first year, degree in Aerospace Engineering, University of Pisa (20 hours).
- "Geometry," first year, degree in Civil Engineering, University of Pisa (40 hours).

2019/2020.

- "Geometry and Linear Algebra," first year, degree in Civil Engineering, University of Pisa (70 hours).
- "Low-Dimensional Topology," third year, Class of Science, Technology and Society, IUSS Pavia (25 hours).

2018/2019.

- "Geometry and Linear Algebra," first year, degree in Civil Engineering, University of Pisa (70 hours).
- "Elements of Mathematics and Statistics," first year, degree in Pharmacy, University of Pisa (26 hours).
- "Knot Theory," Master degree in Mathematics (21 hours).

2017/2018.

- "Geometry and Linear Algebra," first year, degree in Civil Engineering, University of Pisa (90 hours).
- "Elements of Mathematics and Statistics," first year, degree in Pharmacy, University of Pisa (26 hours).

2016/2017.

- "Geometry and Linear Algebra," first year, degree in Civil Engineering, University of Pisa (80 hours).
- "Knot Theory," Master course in Mathematics, University of Pisa (42 hours).

2015/2016.

- "Geometry and Linear Algebra," first year, degree in Civil Engineering, University of Pisa (80 hours).
- "Elements of Algebraic Topology," third year, College course in Mathematics, University of Pisa (48 hours).

2014/2015.

- "Geometry and Linear Algebra," first year, degree in Civil Engineering, University of Pisa (90 hours).
- "Elements of Algebraic Topology," third year, College course in Mathematics, University of Pisa (48 hours).

2013/2014.

- "Geometry and Linear Algebra," first year, degree in Civil Engineering, University of Pisa (90 hours).
- "Elements of Algebraic Topology," third year, College course in Mathematics, University of Pisa (48 hours).

2012/2013.

- "Geometry and Linear Algebra," first year, degree in Civil Engineering, University of Pisa (120 hours).
- "3-manifolds" (in English), second year, Master degree in Mathematics, University of Pisa (42 hours).

2011/2012.

- "Geometry and Linear Algebra," first year, degree in Civil Engineering, University of Pisa (120 hours).

2010/2011.

- "Linear Algebra," first year, degree in Civil Engineering, University of Pisa (60 hours).
- "Geometry," first year, degree in Civil Engineering, University of Pisa (60 hours).
- "Geometry and Linear Algebra," first year, degree in Construction Engineering and Architecture, University of Pisa (80 hours).

2009/2010.

- "Linear Algebra," first year, degree in Civil Engineering, University of Pisa (60 hours).
- "Geometry," first year, degree in Civil Engineering, University of Pisa (60 hours).

2008/2009.

- "Linear Algebra," first year, degree in Civil Engineering, University of Pisa (56 hours).
- "Geometry," first year, degree in Civil Engineering, University of Pisa (56 hours).

2007/2008.

- "Geometry and Algebra, I" , first year, degree in Civil Engineering, University of Pisa (50 hours).
- "Geometry and Algebra, II," second year, degree in Civil Engineering, University of Pisa (50 hours).

2005/2006.

- "Mathematics III," third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- "Linear algebra," first year, degree in Telecommunications Engineering, University of Pisa (50 hours).

2004/2005.

- "Mathematics III," third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- "Linear algebra," first year, degree in Telecommunications Engineering, University of Pisa (50 hours).
- "Curves and surfaces" honor course for students in Engineering, University of Pisa (24 hours).

2003/2004.

- "Mathematics III," third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- "Linear algebra," first year, degree in Telecommunications Engineering, University of Pisa (50 hours).

- "Triangulations, handles, and normal surfaces," doctorate school in mathematics, University of Pisa (20 hours).

2002/2003.

- "Mathematics III," third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- "Geometry and algebra," first year, degree in Telecommunications Engineering, University of Pisa (55 hours).

2001/2002.

- "Mathematics III," third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- "Geometry and algebra," first year, degree in Telecommunications Engineering, University of Pisa (55 hours).

2000/2001.

- "Mathematics III," third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- "Geometry and algebra," first year, degree in Telecommunications Engineering, University of Pisa (55 hours).
- "Complexity and hyperbolic geometry in dimension 3," doctorate school in Mathematics, University of Pisa (40 hours).

1999/2000.

- "Mathematics," third year, degree in Telecommunications Engineering, University of Pisa (100 hours).
- "Geometry and algebra," first year, degree in Telecommunications Engineering, University of Pisa (55 hours).

1998/1999.

- "Foundations of higher geometry I," third year, degree in Mathematics, University of Parma (60 hours).
- "Higher geometry II," fourth year, degree in Mathematics, University of Pisa (course on the interactions between curvature and topology in Riemannian geometry, 40 hours).

- "Geometry and algebra," first year, degree in Telecommunications Engineering, University of Pisa (100 hours).

1997/1998.

- "Geometry II," second year, degree in Mathematics, University of Parma (120 hours).
- "Differential Geometry II," fourth year, degree in Mathematics, University of Pisa (course on the geometric theory of foliations, 40 hours).

1995/1996.

- "Principal bundles and spin structures," doctorate school in Mathematics, University of Pisa (20 hours).

1989/90 – 1998/99. As a graduate student and research assistant I have performed every year support teaching activities for first and second year courses in geometry at the Faculty of Science of the University of Pisa.

4 Dissertations supervised

PhD dissertations:

- Lorenzo Losa, started in 2011, unfinished.
- Valentina Disarlo (joint with Athanase Papadopoulos), started in 2010, defended in 2013.
- Abramo Bertucco, started in 2008, unfinished.
- Vito Sasso (joint with Katya Pervova), *Complexity of links and graphs*, started in 2007, defended in 2011.
- Michele Tocchet (joint with Katya Pervova), *Generalized Mom-structures and volume estimates for hyperbolic 3-manifolds with geodesic boundary and toric cusps*, started in 2007, defended in 2012.
- Fionntan Roukema (joint with Bruno Martelli), *Dehn filling of the minimally twisted* 5*-chain link*, started in 2007, defended in 2011.
- Maria Antonietta Pascali, *Branched covers between surfaces*, started in 2006, defended in February 2010.
- Roberto Frigerio, *Hyperbolic* 3*-manifolds with geodesic boundary*, started in 2001 and defended in March 2005.

- Gennaro Amendola, *Minimal spines and skeleta of non-orientable* 3*-manifolds and bricks*, started in 2000 and defended in January 2004.
- Stefano Francaviglia, *Hyperbolicity equations for cusped* 3*-manifolds and volume-rigidity of representations*, started in 1999 and defended in May 2004.
- Bruno Martelli, *Decomposition into bricks and complexity of* 3*-manifolds*, started in 2000 and defended in February 2002.

Master dissertations:

- Filippo Giovanni Baroni (Pisa), *Realizability of branching data with a short partition*, 2021, approved *cum laude*.
- Filippo Sarti (Pisa), *Combinatorial counting of branched covers between surfaces*, 2017, approved *cum laude*.
- Tom Ferragut (ENS Rennes), *The Hurwitz existence problem*, 2016, approved.
- Erika Pieroni, *Complexity of lens spaces*, 2015, approved *cum laude*.
- Ivo Cicchese, *Surgery presentation of* 3*-manifolds*, 2012, approved *cum laude*.
- Francesca Iezzi, *Open-book decompositions of 3-manifolds and Giroux surfaces*, 2011, approved *cum laude*.
- Lorenzo Losa, *Thurston's hyperbolicity equations*, 2010, approved *cum laude*.
- Sara Maloni, *Teichmüller space for orbifolds*, 2008, approved *cum laude*.
- Vincenzo De Risi, *Geometrization of* 3*-manifolds*, 2006, approved.
- Tiziano Casavecchia, *Finite and ideal hyperbolic polyhedra*, 2005, approved *cum laude*.
- Simone Penzavalle, 2003, *Heegaard splittings of Seifert fibred spaces*, approved *cum laude*.
- Ramona Bassi, 2003, *Two-dimensional orbifolds and the Riemann-Hurwitz formula*, approved.
- Paola Boito, 2001, *Coverings of* 3*-manifolds*, approved *cum laude*.
- Roberto Frigerio, 2000, *Canonical decomposition of hyperbolic* 3*-manifolds with geodesic boundary*, approved *cum laude*.
- Lucio Bedulli, 1999, *Euler structures and Reidemeister torsion*, approved *cum laude*.

- Gennaro Amendola, 1999, *Presentations and calculi in dimension three*, approved *cum laude*.
- Francesco Costantino, 1999, *Canonical decomposition of* 3*-dimensional manifolds*, approved *cum laude*.
- Marina Pescini, 1997, *Ideal cellularizations of hyperbolic* 3*-manifolds*, approved *cum laude*.
- Bruno Martelli, 1996, *Turaev-Viro invariants of* 3*-manifolds*, approved *cum laude*.

Degree dissertations:

- Francesca Pistolato, *Pavimentazioni del piano e orbifold euclidei*, 2018, approved.
- Alessandro Combi, *Mosse tra spine di* 3*-varietà*, 2017, approved.
- Giacomo Puccetti, *The third homotopy group of the* 2*-sphere*, 2016, approved.
- Maurizio Ragusa, *Apparent contours of surfaces*, 2014, approved *cum laude*.
- Luca Pol, *Realization of homology classes*, 2014, approved *cum laude*.

Short (non-research) dissertations:

- Daniele Calbi (Faculty of Engineering), *Lebesgue integral and distributions*, approved.
- Marco Messina (Faculty of Engineering), *Surfaces, curvature, and the Gauss-Bonnet formula*, 2006, approved.
- Stefano Fortunati (Faculty of Engineering), 2005, *Topological surfaces and the coloring problem*, 2006, approved.
- Paola Boito, 2000, *The word problem in the braid group*, approved.
- Sara Biagini, 1999, *Two-dimensional orbifolds*, approved *cum laude*.
- Francesco Costantino, 1998, *Parallelizability of orientable* 3*-manifolds*, approved.
- Francesco Morandin, 1998, *The Kauffman bracket*, approved *cum laude*.
- Nicola Visciglia, 1997, *Tessellations of* 3*-space by knotted tori*, approved *cum laude*.

5 Invited talks and advanced courses

March 31-April 2, 2023, Urbino. *"La storia della matematica in classe,"* conference organized by Centro Matematico PRISTEM, 45-minutes talk on "I grafi da Eulero e i ponti di Kö nigsberg alla teoria moderna."

March 17-19, 2023. *"Knots, Algebra and Geometry,"* Online conference on the occasion of 60th birthdays of Valeriy Bardakov and Andrei Vesnin 40-minutes talk on the solution of the Hurwitz problem with a length-2 partition.

June 23-24, 2021. *ECM8 - Minisymposium "Applied Combinatorial and Geometric Topology,"* 20-minutes talk on the computation of Hurwitz numbers.

February 17-21, 2020. *Winterbraids X*, Centro de Giorgi, Pisa; 3-hours course on the Hurwitz problem.

October 1-5, 2018. *Moscow-Pisa colloquium*, Higher School of Economics and Skoltech, Moscow; 1-hour talk on the computation of Hurwitz numbers.

May 24, 2018. *Colloqui scientifici*, Unione Matematica Italiana, Bologna; 50-minutes talk on advances on the Hurwitz problem.

March 11-14, 2018. *Mini-symposium on computational problems in low-dimensional topology*, OIST, Okinawa 1-hour talk on advances on the Hurwitz problem.

January 25-27, 2018, Paris. *Quand la forme devient substance : puissance des gestes, intuition diagrammatique et phénoménologie de l'espace*, organized by the Collège International de Philosophie. 40-minutes talk on planar diagrams representing topological objects of dimension 1, 2, 3, and 4.

January 26, 2018, Institut Culturel Italien de Paris. *La créativité des diagrammes*, 20-minutes talk on the interplay between mathematics and art from a diagrammatic viewpoint.

June 20-24, 2016. *Quantum topology*, Moscow, 45-minutes talk on advances on the Hurwitz problem.

September 2-9, 2015. 8-hours course on geometric and combinatorial approaches to the Hurwitz realizability problem for branched covers. Financed by the European Community under an Outgoing Erasmus grant. In Heraklion, Crete.

June 2-7, 2013. *Geometric Topology in Cortona*, one-hour talk on extra structures on 3-manifolds via extra structures on spines.

October 12-14, 2012. *Workshop on Teichmüller theory*, Istanbul, one-hour talk on the Hurwitz problem.

July 2-8, 2011. *Knots and Links, from form to function*, ESF-EMS International Conference, Centro De Giorgi, Pisa, 45-minutes talk on Dehn fillings of the 5-chain link.

May, 2011. *Knots and applications*, intensive research trimester, Centro De Giorgi, Pisa, 6-hours course on combinatorial and computer methods in hyperbolic geometry.

June 17-19, 2010. *Computational and Geometric Topology*, Bertinoro. 30-minutes talk on complexity of knots and its additivity properties.

May 1-6, 2009. 5-hours course on *Algorithmic construction and recognition of hyperbolic 3-manifolds*, Master-class in Geometry, Strasbourg.

September 17-20, 2008. *Braids in Paris*, Paris (France). 30-minutes talk on knot and graph complexity.

April 7-11, 2008. After an invitation of J. Andersen, visit to the Centre for the Topology and Quantization of Moduli Spaces (Åarhus, DK), and one-hour talk on the Hurwitz existence problem.

July 6, 2007. After an invitation of Athanase Papadopoulos, visit at the University of Strasbourg and one-hour talk on branched covers between surfaces.

July 2-4, 2007. *Hyperbolic Volume 2007*, Fribourg (CH). One-hour talk on hyperbolic graphs.

June 15-24, 2007. After an invitation of Andrei Vesnin, two-weeks visit at the Sobolev Institute of Mathematics, Novosibirsk, and two one-hour talks on hyperbolic graphs and on manifolds obtained by gluing the faces of the octahedron.

May 30-June 4, 2007. *Troisième Colloque Maghrebien de Geometrie, Topologie et Systèmes Dynamiques*, Marrakech, one-hour talk on branched coverings between surfaces.

May 8, 2007. Talk at the Mathematics Department in Pisa on branched coverings between surfaces.

April 23, 2007. After an invitation of Carlo Pagani, one-hour talk at the University of Milano on 3-manifolds obtained by gluings of the octahedron.

April 16-18, 2007. After an invitation of Dror Bar-Natan, visit to the University of Toronto and one-hour talk on combinatorics and 3-manifold topology.

April 10, 2007. After an invitation of Feng Luo, seminar at Rutgers University on surface branched covers.

April 2007. After an invitation of Dylan Thurston, one-week visit at the Columbia University, New York City, and one-hour talk on low-complexity hyperbolic graphs.

March – May 2007. After an invitation of Maria Rita Casali, series of 5 one-hour talks at the University of Modena on combinatorial and geometric methods in 3-dimensional topology.

March 28, 2007. After an invitation of Salvatore Coen, one-hour seminar at the University of Bologna on branched covers between surfaces.

March 8, 2007. Colloquium talk at the University of Genova on the Poincaré conjecture.

February – March 2007. Series of 5 two-hours lectures at the University of Toulouse on combinatorial approaches to the topology and hyperbolic geometry of knots, graphs, 3-manifolds and orbifolds.

January 25-28, 2007. After an invitation of Wolfgang Metzler, one-hour talk at the University of Frankfurt on hyperbolic knots and graphs.

December 13, 2006. After an invitation of Corrado de Concini, one-hour seminar at the University of Rome I on enumeration of hyperbolic graphs of small complexity.

November 9-23, 2006. After an invitation of Craig Hodgson, two-week visit to Melbourne and one-hour seminar on asymptotic bounds for the complexity of hyperbolic 3-manifolds and groups.

October 14-21, 2006. After an invitation of Athanase Papadopoulos, one-week visit to Strasbourg and one-hour seminar on complexity of hyperbolic graphs.

September 19-22, 2006. Plenary speaker in the conference *Mathematics, Mechanics and Computer Science*, Chelyabinsk (Russia). One-hour talk on hyperbolic knots and graphs.

August 31-September 2, 2006. Invitation to give a one-hour talk on enumeration of knot complexity in the conference *Geometry and Topology of Low Dimensional Manifolds*, Burgo de Osma, Madrid. I actually could not participate due to health problems.

July 3-7, 2006. *Joint meeting of the Unione Matematica Italiana and the Societé Mathématique de France*, Torino, special session on Geometry and Topology of Groups, one-hour talk on enumeration of knotted graphs and 3-orbifolds.

May 22-27, 2006. Invitation to give a 50-minutes talk on enumeration of knotted graphs and 3-orbifolds in the conference *Knot theory, low-dimensional topology and group geometry*, Marseille. I actually could not participate due to health problems.

April 4, 2006. After an invitation of Ian Agol, one-hour seminar at the University of Chicago on complexity of graphs and 3-orbifolds .

April 1-2, 2006. 1015th sectional meeting of the AMS, Miami, special session on Invariants of low-dimensional manifolds, 30-minutes talk on exceptional Dehn surgery.

March 28, 2006. After an invitation of Weiping Li, one-hour seminar at the Oklahoma State University on complexity of 3-orbifolds and knotted graphs.

January 17-February 7, 2006. After an invitation of Craig Hodgson, three-weeks visit to the University of Melbourne and one-hour seminar on branched coverings between surfaces.

January 17-22, 2006. Conference *Manifolds at Melbourne*, one-hour talk on exceptional Dehn surgery.

June 2005. Summer school on the *Topology and geometry of* 3*-manifolds*, ICTP, Trieste; 6-hours course on "Introduction to 3-manifold topology."

April 27-30, 2005. *Seventh panellenic conference on geometry*, Samos; 50-minutes talk on enumeration of 3-manifolds.

April 5, 2005. After an invitation of Joan Porti, visit to Barcelona and 75-minutes seminar on hyperbolic 3-manifolds with geodesic boundary.

February 23-26, 2005. *Workshop on* 3*-manifolds and complexity* in Cortona; one-hour seminar on spherical splitting and complexity of 3-orbifolds.

August 24-26, 2004. *The Keldysh conference* in Moscow; 45-minutes seminar on triangulations of hyperbolic 3-manifolds.

June 1-4, 2004. *Colloque tresses et topologie en petite dimension* in Grenoble; 45-minutes seminar on hyperbolic 3-manifolds with geodesic boundary.

May 6-9, 2004. *INTAS workshop on* 3*-manifolds* in Ebernburg; one-hour seminar on Gadgil's approach to the Andrews-Curtis conjecture; one-hour seminar on the complexity of 3-orbifolds.

May 5, 2004. After an invitation of Wolfgang Metzler, visit to Frankfurt and one-hour seminar on triangulations of 3-manifolds.

September 17-20, 2003. *Colloque Tresses* in Dijon; 30-minutes seminar on exceptional Dehn surgery along knots in handlebodies.

May 2003. Series of 5 lectures of two hours each at the University of Paris VII on various aspects of hyperbolic geometry, enumeration of 3-manifolds, and exceptional Dehn filling.

May 23, 2003. One-hour seminar at the University of South Brittany on the enumeration of hyperbolic 3-manifolds with boundary (invitation by Christian Blanchet).

April 18, 2003. One-hour talk on exceptional Dehn filling of hyperbolic 3-manifolds, within the Special semester *Aspects of real and analytic geometry* at the Centro De Giorgi, Pisa.

September 9-20, 2002. Conference *Advanced course on geometric* 3*-manifolds* (Bellaterra, Barcelona); 50-minutes talk on complexity of hyperbolic 3-manifolds.

June 6-11, 2001. Conference *Tresses* (Luminy, Marseille); 30-minutes talk on hyperbolic 3-manifolds with geodesic boundary.

April 7-9, 2001. Conference *16th British Topology Meeting* (Edinburgh); 40-minutes talk on the complexity of 3-manifolds.

February 15-20, 2001. After an invitation by Wolfgang Metzler, visit to the University of Frankfurt and delivery of two one-hour talks, respectively on the rotation number and Reidemeister-Turaev torsion.

January 29, 2001. After an invitation by Claudio Arezzo, two-hours lecture at the University of Parma on knot diagrams and the rotation number.

November 3-5, 2000. Special session *The topology of* 3*-manifolds* at the 959th Sectional Meeting of the American Mathematical Society (Columbia University, New York City); 20-minutes talk on the complexity of 3-manifolds.

July 2-7, 2000. Conference *Marcel Grossman IX* (Roma), one-hour introductory lecture to hyperbolic geometry and 30-minutes talk on ideal triangulations.

June 28-30, 2000. After an invitation of Theodor Bröcker, visit to the University of Regensburg; two lectures of two hours each on the complexity of 3-manifolds, and one-hour talk on knot diagrams on spines and the rotation number.

September 13-18, 1999. Conference *XVI meeting of the Italian Mathematical Union* (Napoli); 30-minutes talk on degenerate ideal triangulations.

March 16-27, 1998. After an invitation by Michel Boileau, visit to the Université Paul Sabatier (Toulouse) and delivery of three lectures of two hours each, respectively on contact structures, ideal triangulations, and confoliations

September 26-28, 1997. Special session *Non-Euclidean and space-time geometries* at the 924th Sectional Meeting of the American Mathematical Society (Montreal); 30-minutes talk on partially flat ideal triangulations of cusped hyperbolic 3-manifolds.

September 7-13, 1997. Conference *Niedrigdimensionale Topologie* (Oberwolfach); one-hour talk on the construction of tight contact structures via branched spines.

September 3-6, 1997. Conference *New Italian contributions to Differential Geometry* (Bari); 50-minutes talk on tight contact structures and branched spines.

February 1997. Visit to the MSRI in Berkeley; conference *Computational and algorithmic aspects of* 3*-dimensional topology*; one-hour talk on branched spines.

June 3-7, 1996. Conference *Differential geometry and topology* (Palermo); 45-minutes talk on branched spines.

April 19-22, 1996. Conference *V Porto Meeting on Knots and Physics* (Porto); 50-minutes talk on branched spines.

April 1995. Two-weeks visit to the International School of Advances Studies in Trieste, and teaching of a 12-hours graduate course on *Graphic presentations of* 3*-manifolds, Temperley-Lieb algebra, and quantum invariants*.

July 18-22, 1994. Conference *Knots and* 3*-manifolds* (Luminy, Marseille); 30-minutes talk on bracket invariants of 3-manifolds with boundary.

March 1994. One-hour seminar on graphic presentations of 3-manifolds with boundary (Milano, invitation by Maria Dedò).

March 1994. Two 45-minutes seminars on hyperbolization of link complements and Poincaré's polyhedron theorem (Trieste, invitation by Bruno Zimmermann).

January 1994. Two-weeks visit to the ETH in Zürich after an invitation by Bruno Colbois. Six-hours course on *Thurston's hyperbolic Dehn surgery theorem*, and one-hour seminar on Poincaré's polyhedron theorem.

September 1993. Two one-hour seminars on the hyperbolization of link complements (Parma Physics Department, invitation by Enrico Onofri).

April 1990. Two one-hour seminars on the Jorgensen-Thurston theory of hyperbolic 3-manifolds (Parma, invitation by Giuliana Gigante).

6 Conferences attended

February 16 - 20, 2026. *Bridging visualization and understanding in Geometry and Topology* (Paris).

June 13 - 17, 2022. *Quantum Topology and Geometry* (Paris).

July 8 - 12, 2019. *Riemann Surfaces and Teichmüller Theory* (St. Petersburg).

June 11 - 12, 2019. *Topologie de basse dimension* (Paris VII).

July 17 - 22, 2016. *Knots in Hellas 2016* (Ancient Olympia).

July 6 - 10, 2015. *Classical and quantum hyperbolic geometry and topology* (Orsay).

February 4 - 7, 2015. *Applications of Topology* (Lisbon).

May 5-10, 2008. *Invariants in Low-Dimensional Topology* (Oberwolfach).

September 4-8, 2007. *Journées Toulousaines autour des tresses et des nœuds* (Toulouse).

June 28-July 2, 2004. *Fourth European congress of mathematics* (Stockholm).

June 13-16, 2004. *Topology and singularities* (Dijon).

April 1-5, 2003. *Fundamental groups of hyperbolic manifolds* (Les Diablerets, CH).

July 6-12, 2002. *Janos Bolyai conference on hyperbolic geometry* (Budapest).

September 10-15, 2001. *Kleinian groups and hyperbolic manifolds* (Warwick).

June 5-8, 2000. *Journées toulousaines autour des tresses et des nœuds* (Toulouse).

June 21-24, 1999. *Topologie en petite et grande dimension* (Orsay).

June 22-26, 1998. *Low-dimensional topology – The Kirbyfest* (Berkeley).

November 13-15, 1996. *Geometric group theory and low-dimensional topology* (Bochum).

March 4-5, 1996. *Polyhedral approaches to 3-dimensional manifolds* (Paris, IHP).

July 1995. Summer School *Geometry and Physics* (Odense).

July 25-29, 1994. *Geometric methods in low-dimensional topology* (Lyon).

May 19-20, 1994. *Differential Geometry and Complex Analysis* (Parma).

June 20-24, 1990. *International Conference on Topology and its Applications* (Dubrovnik).

7 Administrative and organizational activities

7.1 Main services

December 2023 – June 2026. Member of the National Scientific committee for the Abilitation of associate and full professors in geometry and algebra.

March 2024 – Today. Member of the Board of Directors of "Il Giardino di Archimede, Un Museo per la Matematica" (Ente del Terzo Settore.

June 2021 – October 2022. Institutional coordinator of the proposals submitted by the University of Pisa within Italy's PNRR activities.

July 2019 – October 2022. Vice-rector of the University of Pisa.

November 2016 – July 2019. Member of the Academic Senate of the University of Pisa.

2016-2018. Member of the National Scientific committee for the Abilitation of associate and full professors in mathematical logic and complementary mathematics.

November 2015 – July 2019. Chair of the Department of Mathematics of the University of Pisa.

November 2012 – October 2015. Head of the Bachelor and Master Courses in Mathematics of the University of Pisa.

October 2011 – December 2014. Head of the committee of the University of Pisa for the distribution of research funds to mathematicians and computer scientists.

7.2 Participation to selection and evaluation committees

March-June 2026. Member of the selection committee for tenure-track research position in Geometry, Department of Mathematics, University of Modena and Reggio Emilia.

February 2026. Referee for two projects submitted to Programma per Giovani Ricercatori – Rita Levi Montalcini.

September 2024. Chair of the selection committee for a 2-year postdoc grant in Geometry, Department of Mathematics, University of Pisa.

May 2024. Chair of the selection committee for the PhD school in Mathematics, University of Pisa.

February 2024. Member of the PhD thesis committee for Giovanni Italiano, Scuola Normale Superiore.

December 2023. Member of the PhD thesis committee for Matteo Migliorini, Scuola Normale Superiore.

July 2023. Chair of the selection committee for a 2-year postdoc grant in Geometry, Department of Mathematics, University of Pisa.

June 2023. Chair of the selection committee for the PhD school in Mathematics, University of Pisa.

December 2021 - February 2022. Member of the committee for the selection of a full professor in Geometry, University of Florence.

November 2021. Member of the committee for the upgrade of a researcher in Geometry to the position of associate professor, University of Pisa.

March – July 2020. Member of the selection committee for a position of junior researcher in Geometry in Roma Sapienza.

March – May 2020. Member of the selection committee for a position of full professor of Complementary Mathematics in Pisa.

August – September 2019. Member of the evaluation committee for admission to the Scuola Normale Superiore di Pisa.

July – September 2019. Member of the selection committee for a position of associate professor of Geomety in Modena.

July – September 2019. Member of the selection committee for a position of full professor of Geometry in Roma II.

May – June 2019. Member of the selection committee for a position of associate professor of Complementary Mathematics in Pisa.

July – October 2017. Member of the selection committee for a position of associate professor in geometry at the Department of Mathematics in Florence.

September 2017. Head of the selection committee for a one-year post-doc position in mathematical analysis at the Department of Mathematics in Pisa.

September 2017. Head of the selection committee for a one-year post-doc position in ergodic theory at the Department of Mathematics in Pisa.

September 2017. Head of the selection committee for a one-year post-doc position in probability at the Department of Mathematics in Pisa.

March 2016. Head of the selection committee for a one-year post-doc position in probability at the Department of Mathematics in Pisa.

January 21-22, 2016. Member of the Evaluation Committee (comité d'experts) for HCERES of the Mathematics Laboratory of the University of Clermont-Ferrand.

January – February 2016. Head of the selection committee for a tenure-track associate professorship in Algebra and Geometry at the University of Pisa.

January 2016. Head of the selection committee for a one-year post-doc position in Geometric Topology at the University of Pisa.

Summer 2015. Member of the selection committee for a temporary research fellowship at the University of Milano.

February 2015. Member of the selection committee for an associate professor in geometry at the University of Pavia.

November 2014. Member of the selection committee for two associate professors in geometry at the University of Pisa.

July 2014. Head of the committee for the selection of an associate professor in Geometry at the University eCampus.

December 2011. Head of the committee for the selection of a permanent research fellow in Geometry at the University of Pisa.

2010. Member of the selection committee for a position of full professor in Geometry, University of Padova.

February 2010. Member of the selection committee for the Junior Fellowship Positions at the Centro de Giorgi, Scuola Normale Superiore, Pisa.

February 2010. Member of the evaluation commitee for AERES of the Institut Fourier in Grenoble.

September 2008. Member of the evaluation committee for admission to the Scuola Normale Superiore di Pisa.

November 2007. Member of the evaluation committee for admission to the Doctorate School in Mathematics, University of Pisa.

September 2009. Member of the selection committee for admission to the PhD in Mathematics at the Scuola Normale Superiore di Pisa.

September 2004. Member of the evaluation committee for admission to the Scuola Normale Superiore di Pisa.

November 2003. Member of the evaluation committee for admission to the Doctorate School in Mathematics, University of Pisa.

7.3 Organization of scientific events

September 5-9, 2023, Pisa. Member of the organization committee of *XXII Congresso dell'Unione Matematica Italiana*.

June 29-July 2, 2021, Pisa. Member of the scientific committee of the meeting *Real and Complex Manifolds, the mathematical heritage of Edoardo Vesentini*.

January 14-17, 2019, Pisa. Co-organizer of the meeting *Alessio Figalli Fields medallist 2018*.

September 17-20, 2018, Wroclaw. Co-organizer of the special session *Geometric Topology, Manifolds and Group Actions* within the first joint meeting of the Polish and Italian mathematical societies.

December 2017-May 2018. Co-organizer with Luciano Modica of the cycle of seminars "Il teorema più bello". This activity also lead to the publication [67].

May 2, 2017. Co-organizer with Fabrizio Broglia and Riccardo Benedetti of a day on algebraic and analytic geometry in honor of Margherita Galbiati.

August 29 – September 2, 2016. Co-organizer with Daciberg Goncalves (Sao Paulo), Isaia Nisoli (Rio), Sergio Martins (Sao Paulo), and Stefano Francaviglia (Bologna) of the special session "Geometric Topology and Dynamics" within the first joint meeting of the Italian and Brazilian mathematical societies, in Rio.

June 30 – July 2, 2014. Co-organizer with Joan Porti (UAB) of the special session "Geometric-Topology" within the first joint meeting of the Italian and Spanish mathematical societies, in Bilbao.

May – July 2011. Member of the organizing and scientific committees of an intensive trimester on "Knots and their applications" at the Centro De Giorgi, Pisa, with Renzo Ricca, De Witt Sumners, Ken Millet, Louis Kauffman and Dale Rolfsen.

May 20-25, 2007. Joint organizer with Riccardo Benedetti, Roberto Frigerio, Giovanni Gaiffi, and Mario Salvetti of the international meeting *Braids and their Ramifications* in Cortona. The schedule included talks by about 30 renowned experts in the algebraic and topological branches into which the theory of braids has evolved.

June 6-24, 2005. Joint organizer with Michel Boileau and Bruno Zimmermann of an activity entitled *Geometry and topology of* 3*-manifolds* at the International Center for Theoretical Physics (ICTP) in Trieste. The activity included two weeks of summer school, particularly addressed to pupils coming from developing countries, and a week-long conference with over 130 participants and 32 talks, given by several of the most prominent researchers in the subject. The ICTP has financed this activity with a budget of Eur. 78 000.

February 23-26, 2005. Organizer of an INTAS workshop in Cortona on the topology and complexity of 3-manifolds. The activity involved about 30 mathematicians from many different countries and included 14 one-hour seminars over 5 days, together with many informal sessions of joint research.

June 19-22, 2002. Joint organizer with Riccardo Benedetti and Mario Salvetti of the international meeting *Braids in Cortona*. The schedule included 4 mini-courses and 20 talks, and about 90 participants attended the conference.

June 11-16, 2002. Joint organizer with Riccardo Benedetti, Jeff Weeks, and Dale Rolfsen of a special session on *The topology of* 3*-manifolds* within the First Joint Meeting of the American Mathematical Society and the Unione Matematica Italiana, in Pisa. A follow-up informal session on June 17 under the same title was also organized. Twenty talks were delivered and about 50 more participants attended the event.

1999/2000. Joint organizer with Riccardo Benedetti and Paolo Lisca of a special activity on *Geometric and Symplectic Topology* in Pisa, with one- and two-week visits by A. Stipsicz, D. Kotschick, F. Laudenbach, D. Auroux, L. Kauffman, M. Hutchings, J.-C. Sikorav, R. Piergallini, J. Weeks, B. Zimmermann, M. Reni, S. Vidussi, L. Funar.

7.4 Direction of research projects

January 2009 - December 2010. Responsible of an Azioni Integrate Pisa-UAB (Barcelona) activity entitled "Geometry of groups and manifolds," funded with Eur 11 280 .

January 2007 – December 2008. Scientific and administrative coordinator of a research project entitled "Combinatorial methods based on triangulations in low-dimensional geometric topology and geometric group theory" within the framework of the joint Italian CNR and French CNRS activities. The project, which included teams from Pisa and from Strasbourg, was approved and granted financial support up to a maximum of Eur 36 000.

January 2007 – December 2008. Scientific and administrative supervisor of a Marie Curie activity entitled "Complexity and quantum invariants of 3-manifolds and knots." The activity included a fellowship granted to Ekaterina Pervova at the Department of Applied of Mathematics of the University of Pisa, and it was funded by the EU with a total sum of Eur. 117 300.

March 2004 – February 2007. International coordinator of an INTAS project entitled "Complexity, algorithms, and computer methods in geometric topology." This project was funded by INTAS with Eur. 110 000 and included scientific teams from Pisa, Frankfurt, Novisibirsk, Chelyabinsk.

February 2004 – July 2005. Scientific and administrative supervisor of a Marie Curie activity entitled "Use of combinatorial methods in various approaches to three-dimensional topology." The activity included a fellowship granted to Gwénaël Massuyeau at the Department of Applied of Mathematics of the University of Pisa, and it was funded by the EU with a total sum of Eur. 103 000.

7.5 Other administrative offices

2026. Referee for a paper submitted to Journal of the European Mathematical Society (JEMS).

2025. Referee for papers submitted to Discrete and Computational Geometry, Journal of Knot Theory and its Ramifications, Journal of the European Mathematical Society (JEMS), Mathematische Zeitschrift, Journal of Geometric Analysis.

2024-2026. Individual member delegate of the EMS Council.

2024. Referee for a paper submitted to the Algebraic and Geometric Topology.

2023. Referee for a paper submitted to the Pacific Journal of Mathematics.

2020-2022. Individual member delegate of the EMS Council.

2022. Referee for a research project submitted to the Natural Sciences and Engineering Research Council of Canada and to the Luxembourg National Research Fund (FNR).

2021. Referee for papers submitted to Publicationes Mathematicae Debrecen and C.R. Comptes Rendus Mathématiques de l'Académie des Sciences.

2020. Referee for papers submitted to the Bulletin of the London Mathematical Society.

May – June 2019. Member of a Focus group in charge of analysing the relations between the central administration and the research and teaching structures of the University of Pisa.

2019. Reviewer of research projects for the Fonds National del la Recherche Luxembourg.

February 2019 – today. Expert (*esperto disciplinare*) for the Italian evaluation agency of universities and research institutions (ANVUR).

2017. Referee for papers submitted to Algebraic and Geometric Topology.

2016. Referee for papers submitted to Annali della Scuola Normale Superiore, Journal of Group Theory, Proceedings of the American Mathematical Society.

2016. Reviewer of research projects for the Fonds National del la Recherche Luxembourg.

2015. Referee for papers submitted to Algebraic and Geometric Topology, Annales de la Faculté de Science de l'Université de Toulouse, Proceedings of the American Mathematical Society (twice).

2014. Referee for papers submitted to Quantum Topology, Annali della Scuola Normale Superiore, Algebraic and Geometric Topology.

2010. Referee for papers submitted to Pacific Journal of Mathematics, Quarterly Journal of Mathematics, Central European Journal of Mathematics, Mathematische Annalen, Proceedings of the American Mathematical Society, Journal of Knot Theory and its Ramifications.

2009. Scientific and technical responsible for the creation of the online edition of the Annali della Scuola Normale Superiore, Classe di Scienze.

2009. Referee for papers submitted to Journal of Knot Theory and its Ramifications, Journal für reine und angewandte Mathematik, Ars Mathematica Contemporanea.

2008. Referee for papers submitted to Algebraic and Geometric Topology, Topology and its Applications, Advances in Mathematics, Pacific J. Math.

2008. Referee of a research project submitted to the CNRS (France).

December 2007. External referee and member of the evaluation committee for two doctorate theses at the University "La Sapienza," Rome

August 22-27, 2005. Member of the scientific committee of the *Workshop on geometry and topology of 3-manifolds* held in Novosibirsk.

April 2005-December 2006. Representative of the Department of Applied Mathematics in the steering board of the Doctorate School in Mathematics of the University of Pisa.

2006. Referee for papers submitted to Geometry and Topology, Geometriae Dedicata, Annali della Scuola Normale Superiore di Pisa.

April – June 2005. Member of a committee of the Department of Applied Mathematics for a global reorganization and revision of the contents of the mathematics courses taught at the Faculty of Engineering of the University of Pisa.

April 26, 2005. External referee and member of the evaluation committee for the doctorate thesis of Guillaume Théret, Université Louis Pasteur, Strasbourg

2005. Referee for papers submitted to Commentarii Mathematici Helvetici, Acta Applicandae Mathematicae, and Rendiconti del Circolo Matematico di Palermo

1994 – today. Reviewer of papers for Mathematical Reviews (84 papers reviewed so far).

1999 – 2012. Master of the web page of the research group in topology and geometry of the Department of Mathematics in Pisa.

2004. Referee for papers submitted to Advances in Geometry, Acta Applicandae Mathematicae, and Edizioni dell'Unione Matematica Italiana.

2002/03 – 2003/04. Elected as a representative of full professors in the permanent committee of the University of Pisa for the assignment of research funds in Mathematics and Computer Science.

2003. Referee for papers submitted to the Pacific Journal of Mathematics, the Bulletin of the Mathematical Society of Mexico, Advances in Geometry, the International Journal of Mathematics and Mathematical Sciences, and Experimental Mathematics.

2002. Referee for papers submitted to Geometriæ Dedicata and Mathematisches Nachrichten.

January 2002. External referee and member of the evaluation committee of the PhD dissertation in Mathematics of Guzman Tierno at the University of Rome I.

2001. Referee for papers submitted to Experimental Mathematics, Rendiconti dell'Accademia Nazionale dei Lincei and Atti dell'Accademia dei XL.

1996/97 – 1998/99. Organizer of the visits to Pisa of É. Giroux, J. Weeks, J. Porti, M. Boileau, S. Matveev, D. Repovs, Y. Kawahigashi, I. Murakami, A. Mednykh.

1996/1997 and 1997/1998. Member of the governing board of the Department of Mathematics, University of Pisa.

Spring 1995. Member of the organizing committee of an intensive semester on *Low dimensional topology* in Pisa, with talks and courses by M. Boileau, J.-P. Otal, A. Marin, É. Giroux, D. B. A. Epstein, F. Paulin, V. Turaev, G. Masbaum.

1994/95 – 1997/98. Elected as a representative of research assistants in the permanent committee of the University of Pisa for the assignment of research funds in Mathematics and Computer Science.

1987/88 and 1988/89. Elected as a representative of students in the executive council of the Scuola Normale Superiore.

8 General information

Citations

According to Mathematical Reviews, as of March 2026 my papers and books have been cited by 1253 other papers. Excluding the textbook [4], all citations refer to research work, and they total to 760. My h-index is 18. The same figures according to Google Scholar are 3017 total citations, 1670 citations to research work, h-index 25.

Computing skills

I write my papers using LaTeX,of which I have grown rather proficient, and I produce my pictures using CorelDraw (more recently, IPE).

While working on Thurston's solitaire tilings of the plane [7] I have used Mathematica and written some code to do the relevant computations and draw the pictures of the tilings.

I am running my own web page and until 2012 that of the geometry and topology re-

search group in Pisa, using the HTML language. I have considerably contributed to the development of the Web page of the Annali della Scuola Normale Superiore, Classe di Scienze, and I constantly work to update the information it contains.

I have carried out part of the hyperbolic graph census [46], and the enumeration of octahedral manifolds [50], using Haskell, a purely functional programming language that I have come to master rather deeply.

Language skills
Besides Italian, I speak English and French fluently, and I have given many talks and lectures in these languages. I understand Spanish and I speak the very basics of it.

Editorial expertise
Having been since 1992 the main Secretary of the Editorial Board of a Scientific Journal, working under 4 different directors and managing its transition from the paper-only to the on-line version, I can claim I have a very deep experience and knowledge of the scientific editorial business.

Hobbies
I like sports and outdoor activities. I regularly practice long-distance and trail running, scuba diving, downhill skiing, and less regularly road and mountain biking, canoeing, mountain hiking, gym fitness.

9 Scientific activity: completed works

1988-1990: holomorphisms of domains in complex Banach spaces My research activity started under the supervision of Edoardo Vesentini, with an internal seminar at the Scuola Normale and the degree dissertation. I obtained generalizations to various situations of classical non-homogeneity theorems valid for domains in complex two-dimensional space. My results on this topic are collected in three short papers [1, 2, 3].

1990-1992: hyperbolic geometry At the beginning of my PhD studies I switched from complex analysis to the somewhat different subject of hyperbolic geometry. For quite some time I mostly concentrated on the joint writing with Riccardo Benedetti of a graduate textbook on this topic, later published by Springer-Verlag [4]. None of the results our book contains is strictly speaking new, but we have often reworked and simplified the known proofs, sometimes filling gaps and making the arguments more easily accessible. The response to our book has been very favourable worldwide, and the book was reprinted in 1996 and 2003.

1991: hyperbolicity equations Starting from the book written with Benedetti, my mathematical interests have been focused on low-dimensional topology and geometry, and particularly on the theory three-dimensional manifolds, where hyperbolic geometry plays a central rôle. My first original contribution [5] to this field illustrates a very flexible method to find a (topological) ideal triangulation of a link complement in the 3-sphere, and hence to investigate hyperbolicity of the link. I mention here that years later I have extended the method to complete generality [10].

1992: Poincaré's polyhedron theorem As a PhD student I have spent the academic year 1991/92 in Warwick, working with David Epstein on Poincaré's polyhedron theorem, which gives a powerful method for constructing hyperbolic, Euclidean, and elliptic manifolds and orbifolds. We have given a unified proof for all dimensions and geometries, particularly focused on the algorithmic aspects. Our result is published in [6].

1992: tessellations of the plane In Warwick I also worked on a method suggested by Thurston for constructing self-similar tessellations of the plane. This method involves notions coming from theoretical computer science (the theory of finite state automata) and from number theory (the theory of Pisot numbers) and leads to tessellations having tiles of fractal shape. I systematized the ideas of Thurston and produced many examples, publishing my results in [7].

1993-1994: standard spines and quantum invariants After returning from Warwick I resumed my collaboration with Benedetti. We devoted our attention to the theory of standard spines, with the idea of funding a new combinatorial approach to 3-manifolds. Our hope was to apply the combinatorial framework to the investigation of the various topological and geometric theories of 3-manifolds that were spreading at that time, in particular the theory of quantum invariants. As a first step in this project we have introduced a convenient graphic encoding for standard spines and for the moves of the Matveev-Piergallini calculus on spines. This encoding has suggested us the (formal) definition of a class of invariants, called bracket invariants, that contains all the Turaev-Viro invariants. Our results were published in [8]. They also appear in my PhD thesis [9], together with the result already mentioned on ideal triangulations of link complements, published separately in [10]. As a first application of our combinatorial encoding of 3-manifolds we have proved an extension to the case of non-empty boundary of the Turaev-Walker theorem, according to which each Turaev-Viro invariant is just (up to normalization) the squared module of the corresponding Reshetikhin-Turaev-Witten invariant. This result was published in [11].

1994-1995: branched standard spines The next applications of the combinatorial approach of Benedetti and me to spines and 3-manifolds, after quantum invariants, was obtained, starting from 1994, by investigating the relation with the notion of branched surface. It turned out that when a polyhedron is at the same time a branched surface

and a standard spine of a 3-manifold, there is a naturally induced geometric structure on the manifold. The rich theory arising from this observation was illustrated in a monograph [12], and in the proceedings of a conference [13]. We have been able to provide combinatorial presentations, according to the objects/moves scheme popular in dimension three, of the following topological categories: combed manifolds, framed manifolds, and spin manifolds. Some remarkable connections of our approach with the theory of contact structures and foliations were also pointed out in the monograph.

1996: contact structures After the publication of the monograph [12], Benedetti and I have concentrated on the relations between our results and tight contact structures, at that time one of the most mysterious and challenging topics in low-dimensional topology. We have shown that under certain circumstances a branched spine can "carry" a tight structure, and we have proved notable uniqueness results, coming from the intrinsically global nature of the topological information provided by both a tight contact structure and a standard spine. Our results appeared in [14].

1996: topology textbook In 1996 I wrote with Chiara de Fabritiis an advanced undergraduate textbook [15] of problems (with solutions) in topology and geometry. We have included short accounts of the theoretical results needed to face the problems, and the detailed solution of 200 problems. The following topics are covered in the book: general topology, homotopy and homology theory, algebraic curves, complex analysis, differential geometry of curves and surfaces, classical groups.

1996-1997: confoliations At the end of 1996, jointly with Riccardo Benedetti and Paolo Lisca, I organized a reading seminar in Pisa on a paper (then a preprint) of Eliashberg and Thurston on certain relations between contact structures and foliations, the theories of which were unified under the new concept of *confoliation*. The main result of their paper is quite astonishing, but many details of the proof were not completely clear. Thanks to our joint work in the seminar and to several conversations I had with Thurston, I was eventually able to write down a self-contained and detailed proof, published in [16]. The main point I added to the argument is a technical result on the holonomy of codimension-1 foliations, valid in any dimension, published separately in [17].

1997: flat hyperbolic triangulations During 1997 I worked with Jeff Weeks on geodesic ideal triangulations of cusped hyperbolic three-manifolds. Existence of these triangulations is commonly believed and experimentally true, but not proved in general. Existence is however known if one allows the triangulation to include some flat tetrahedra. Having this fact in mind as a motivation, we have investigated the geometric properties of partially flat solutions of hyperbolicity equations. We have obtained the surprising result that consistency equations alone are not sufficient, in the partially flat case, to ensure existence of an (incomplete) hyperbolic structure, because topology can degenerate.

However, as one could reasonably expect, if also the completeness equations are satisfied then the solution does provide a hyperbolic structure. Our paper appeared in [18].

1998: hyperbolic Dehn filling via inverted triangulations In the Spring of 1998 I started collaborating with Joan Porti on a proof of Thurston's hyperbolic Dehn filling theorem, based on the technology of ideal triangulations only, and without the assumption of existence of a genuine triangulation with all positive-volume tetrahedra. We have employed a particular partially flat triangulation obtained as a subdivision of the canonical Epstein-Penner cellularization. Using also the techniques developed by Weeks and me, we have been able to provide a completely self-contained and comparatively elementary proof of the filling theorem. In particular, we have given a new proof of the fact that the hyperbolic filling coefficients constitute an open set, without assuming smoothness of the complete point in the variety of deformations. Smoothness is known within the approach through representations of the fundamental group, but the existing proofs of smoothness in the context of triangulations appear to be incomplete. Our paper was published as [19].

1998-2000: Reidemeister-Turaev torsion Starting from 1998 Benedetti and I have been working on a refinement of an invariant (due to Turaev) that, in a suitable sense, lifts the Reidemeister torsion. Turaev's torsion was originally defined for "Euler structures," that is for non-singular vector fields viewed up to homotopy and modifications supported within balls, on manifolds that are closed or bounded by tori, with the extra requirement that the field should never be tangent to the boundary. After several partial results and improvements, which caused two revisions of the first August 1998 preprint, the last of which dated February 2000, we have been able to extend the definition of torsion, and the fundamental H_1-equivariance property, to Euler structures having arbitrary tangency lines to the boundary. The key technical tools of our refinement are the extension of the classical Poincaré-Hopf theorem to fields with boundary tangency and the definition of the combinatorial counterpart of a smooth Euler structure. Two more ingredients are however crucial, namely the proof that torsion can be computed directly from a branched spine, and the remark that our generalized torsions apply in particular to pairs field-link such that the link is transversal to the field. In this context we have also shown that, for a knot in a homology sphere, our torsion contains a lifting of the classical Alexander invariant. Our paper appeared in [20].

1999: fatherhood probability During 1999 I cooperated with Nicola Cucurachi, a research assistant in Legal Medicine at the University of Parma. We have jointly authored a paper about paternity tests through genetic analysis. I have been often called by Cucurachi for the solution of probability problems arising in this type of tests, and I have observed that medical papers often contain errors, sometimes gross ones. Our paper tries to put straight some of the points most often wrongly addressed in the literature. The mathematical content of the paper is of course very elementary, but the effort of mutual understanding between languages and cultural environments as far from each other as

Mathematics and Medicine was quite stimulating for me. Our paper appeared in [21].

1999: degenerate hyperbolic triangulations At the end of 1999 I collected in a new article the main points of the papers with Weeks and Porti. I concentrated on the questions remained open, addressing in particular the (well-known and hard) problem of a general understanding of the topological and geometric meaning of a degenerate (partially inverted) solution of hyperbolicity equations. This article contains some new results and it was presented in a seminar I gave at a conference. It appeared in [22].

1999-2000: Euler structures with boundary and framed knots Between the end of 1999 and the beginning of 2000, stimulated by our investigations on Reidemeister torsion for Euler structures, Benedetti and I have succeeded in extending our calculus for combed manifolds from the closed to the bounded case. We have also provided a combinatorial encoding for pairs (non-singular field)–(transversal link) up to homotopy of the field and transversal isotopy of the link. These results, together with the refinement of Turaev's calculus for framed knots represented by diagrams on branched spines, have allowed us to provide an extension of the notion of 'winding number' of a knot diagram. A remarkable fact is that we can define this number not only when the ambient manifold is a homology sphere, but also for most knots having hyperbolic complement. This paper appeared in [23].

1999-2000: Feynman diagrams and topology In the Summer of 1999 I started working with Roberto De Pietri, a physicist in Parma, on the topological interpretation of the Feynman diagrams arising from the expansion of a certain partition function, proposed by De Pietri himself with Rovelli, Krasnov, and Freidel. From the mathematical viewpoint the problem reduces to the issue of checking whether certain simplicial 4-complexes are 4-manifolds with boundary. In general this issue contains the (very hard) problem of recognizing the 3-sphere, but the 4-complexes arising in the physical setting were actually simple enough that we could give a definite and algorithmic answer to the problem. Our work appeared in [24].

1999-2000: enumeration of 3-manifolds In the Fall of 1999 I started collaborating with my graduate student Bruno Martelli on a project of enumeration of 3-manifolds of small complexity. I am referring here to Matveev's complexity, defined by means of standard spines and equal, for irreducible manifolds, to the minimal number of tetrahedra in a triangulation. Martelli carried out extensive computer experiments, and the analysis of the data he collected suggested us a very powerful structure theorem for 3-manifolds, later proved in full generality in the framework of a new theory of complexity. Namely, we have shown that there exist certain manifolds with (suitably marked) toric boundary, that play the rôle of elementary "bricks," in the sense that all 3-manifolds are obtained by (suitably) assembling these manifolds. What is actually most remarkable (and very

important from the computational viewpoint) is that, in any given complexity, the number of bricks is dramatically smaller than the number of all manifolds. For instance, there are 1155 manifolds in complexity 9, and (essentially) 7 bricks only are already sufficient to generate all of them. Another notable feature of our decomposition is that all the bricks we have found so far are geometrically atoroidal. This implies that, for manifolds obtained from these bricks only, our decomposition is actually a refinement of the celebrated Jaco-Shalen-Johannson decomposition, that provides the topological starting point of Thurston's geometrization conjecture. Our theoretical and experimental results appeared in [25].

2000-2001: non-orientable 3-manifolds and bricks Between the end of 2000 and the beginning of 2001 I have continued working with Bruno Martelli on the complexity of 3-manifolds, proving the non-orientable analogue of the decomposition theorem into bricks mentioned above. Extending a result from the orientable to the general case is sometimes straight-forward or even tedious, but this was definitely not true in our case. We have observed in particular that accepting Klein bottles on the boundary entails a variety of new and very interesting topological phenomena, that have no orientable counterpart. In particular, finiteness of the number of irreducible manifolds in any given complexity is only true in a more refined sense than in the orientable context. This has forced us to give a somewhat more accurate definition of the notion of brick, and to develop new techniques for the proof of the decomposition theorem. Our paper was published in [26].

2000: combinatorial Reidemeister torsion In the Fall of 2000 Benedetti and I have been working with our students Gennaro Amendola and Francesco Costantino on a new purely combinatorial approach to the theory of the Reidemeister-Turaev torsion, using our calculi for manifolds and knots. This approach has the advantage of allowing to define torsion in situations more general than the "classical" ones already considered in [20]. It also has the drawback that the topological information carried by torsion in the classical cases —but not in the closed case— is weaker. Our paper appeared as an invited one in a special volume [27].

2001: hyperbolicity equations with boundary Starting from the Spring of 2001 I have been working with my student Roberto Frigerio on partially truncated ideal triangulations of hyperbolic 3-manifolds with geodesic boundary. The theory of these manifolds and their triangulations, which has at its core Kojima's result on the existence of a canonical decomposition, is particularly rich when both the manifold and its boundary are non-compact. We have extended the theory of moduli and hyperbolicity equations to this setting, discovering in particular a rather surprising interpretation to give to the notion of ideal triangulation in the case of non-compact boundary. We have also generalized and adapted a procedure (originally described by Weeks in the cusped case) for the transformation of an arbitrary decomposition into the canonical one, using the so-called tilt formula already established by Ushijima. Our results were published in [29].

2001-2003: exceptional Dehn surgery on the 3-chain link The decomposition theory developed by Martelli and me involves four bricks having complexity 9. Two of them have as underlying space a well-known manifold, called the "magic" one by Cameron Gordon, which is the complement in the 3-sphere of a 3-component link whose components are individually unknotted. The other two bricks of complexity 9 are obtained by hyperbolic Dehn surgeries along this link. Using the technology of spines and skeleta we have then been able to completely classify the (infinitely many) non-hyperbolic surgeries along this link (not just the slopes, also the resulting manifolds). Analyzing the nature of these surgeries we have been able to show that most known upper bounds between different types of exceptional fillings on hyperbolic manifolds are realized by the "magic" one. The first 2002 version of our preprint underwent a substantial revision in 2003 and was eventually published in [30].

2002-2003: complexity of geometric 3-manifolds Again with Martelli I have been working, starting from the Fall of 2002, on upper bounds on the complexity of general 3-manifolds obtained by our decomposition theorem into bricks. The idea is that the bricks we already have, even if certainly not sufficient to construct all hyperbolic manifolds, already allow to construct huge classes, including all Seifert fibered spaces and torus bundles, and therefore provide upper bounds for the complexity of these manifolds. With a careful search for the "best" realization via the bricks known so far, we have actually found a precise formula for a certain "approximated complexity" for Seifert spaces and torus bundles [31] . We actually conjecture this formula to give the exact value of complexity except for some very special atoroidal Seifert manifolds. This conjecture extends and unifies previous ones of Matveev and Anisov.

2002-2004: enumeration of hyperbolic manifolds with boundary Building on one side on the theoretical results [29] on hyperbolicity equations and the tilt formula, and on the other side on the computer programs already used in the closed case [25], I have developed with Frigerio and Martelli a software which has enabled us to list hyperbolic manifolds with geodesic boundary up to complexity 4. We have confined ourselves to manifolds having non-empty *compact* boundary, thus avoiding annular cusps, but we have allowed toric cusps. After reproducing the known census in complexity 2, which comprises 8 compact manifolds, we have found 151 in complexity 3 (one of which has a toric cusp) and 5033 in complexity 4 (30 having one toric cusp and one having two). For all the manifolds in the census, using the tilt formula and a volume formula of Ushijima, we have computed the Kojima canonical decomposition and the volume, discovering many remarkable combinatorial and numerical data. Our results were published [33] by a specialized journal. A systematic approach to the enumeration issues, which includes the case of manifolds with non-compact boundary, was provided in [38].

2002: exact complexity computations The experimental data found in [33] suggested Frigerio, Martelli and me to consider the special class $\mathcal{M}_n$ of 3-manifolds with

boundary of genus n which can be ideally triangulated with n tetrahedra. We have then shown that the elements of $\mathcal{M}_n$ have complexity n and Heegaard genus $n+1$. Moreover, we have proved that they are all hyperbolic with geodesic boundary, that they all have the same volume, and that their ideal triangulation with n tetrahedra actually is the Kojima canonical decomposition. This last property implies that different ideal triangulations give different elements of $\mathcal{M}_n$. Using this fact and some combinatorics of standard spines, we have shown that $\#(\mathcal{M}_n)$ grows exponentially with n, so indeed this class $\mathcal{M}_n$ is rather large. Our paper [32] describes all these results.

2003: exceptional fillings of large manifolds In another paper [34] completed in the Spring of 2003 and accepted shortly after submission, Frigerio, Martelli, and I have pursued our analysis of hyperbolic 3-manifolds, introducing the class $\mathcal{M}_{g,k}$ of 3-manifolds M such that ∂M consists of one surface of genus g and k tori, and M can be ideally triangulated with $g+k$ tetrahedra. We have then extended to $\mathcal{M}_{g,k}$ the results proved in [32] for $\mathcal{M}_n$, namely that $M \in \mathcal{M}_{g,k}$ has complexity $g+k$ and Heegaard genus $g+1$, that M is hyperbolic with geodesic boundary and k toric cusps, that the volume of M is a function of g and k only, and that the triangulation of M with $g+k$ tetrahedra is Kojima's canonical decomposition, so it is unique. We have then shown that $\#(\mathcal{M}_{g,k})$ grows factorially in g when k is fixed (thus improving the exponential growth estimate established in [32]). Moreover we have completely analyzed the exceptional Dehn fillings of the cusps of the elements of $\mathcal{M}_{g,k}$. This has enabled us, among other things, to prove a conjecture of Wu on the largest possible distance between an annular and a boundary-reducible Dehn filling of a "large" hyperbolic 3-manifold, where "large" means that the homology is not that of a link complement.

2003-2004: geometric investigation of triangulations In a paper [35] completed in February 2004 with Costantino, Frigerio, and Martelli, we have analyzed the set of combinatorial triangulations in geometric terms. More precisely, we have shown that one can canonically associate to a triangulation T a relative hyperbolic handlebody $N(T)$ (*i.e* a pair (H,Γ), where H is a handlebody, Γ is a set of curves on ∂H, and $H \setminus \Gamma$ is hyperbolic with totally geodesic boundary), and a cusped hyperbolic manifold $D(T)$, which actually is a link complement in a connected sum of copies of $S^1 \times S^2$. Moreover, using the canonical decompositions of Epstein-Penner and Kojima, we have proved that the geometries of $N(T)$ and $D(T)$ both determine T. As applications of our constructions we have shown that:

- a triangulation (which may have multiple and self-adjacencies, finite and/or ideal vertices) is determined up to isotopy by its 1-skeleton;
- a triangulation with all edges having valence at least 6 defines a hyperbolic structure on which all the edges of the triangulation can be isotoped to geodesics;
- there exists a constant $c > 0$ such that any finite group G is the full isometry group of a hyperbolic manifold M with $\mathrm{vol}(M) < c \cdot |G|^9$.

2004: complexity of 3-orbifolds In the Spring of 2004 I started working on a generalization to 3-orbifolds of Matveev's complexity theory for 3-manifolds, based on special spines. After giving a rather natural definition of complexity, I succeeded in proving the orbifold analogues of several of the main results valid for manifolds [37]. In particular, I showed that for any n there is only a finite number of irreducible orbifolds up to complexity n. I also proved that, with some well-understood exceptions, the minimal spines of these orbifolds are special, whence dual to triangulations, which shows that the definition is very natural. These results prompt for computer investigation of orbifolds, which I am planning to pursue in the future. Concerning the additivity of complexity under connected sum, which holds for manifolds, I was able to prove it under ordinary and cyclic connected sums. I also proved that additivity cannot hold in a strict sense for vertex connected sum, but I established 2-sided linear estimates on the behaviour of complexity under this operation.

2004: spherical splitting of 3-orbifolds While working on additivity of complexity, I realized that no satisfactory account of the orbifold analogue of the Haken-Kneser-Milnor theorem existed in the literature. This theorem asserts that every 3-manifold splits as a connected sum of prime ones, and the summands are unique provided none of them is the 3-sphere. The orbifold analogue was believed to be true for many years, but it was neither clearly stated nor correctly proved. And it turned out that neither uniqueness nor existence of the splitting are true if one takes the straight-forward generalization, which excludes *tout-court* the spherical 3-orbifolds as possible summands, because, in a suitable sense, the operations of connected sum of different types do not commute with each other. Even after overcoming this problem of non-commutativity, existence itself of the splitting remains not clear when the orbifold contains non-separating spherical 2-orbifolds. Moreover existence cannot be proved using maximal essential systems of spherical 2-orbifolds, as one does in the manifold case. Restricting to orbifolds which contain no bad 2-suborbifolds and in which every spherical 2-orbifold is separating, I have then given a precise statement of the splitting theorem [36], also providing a detailed proof of existence and uniqueness, which uses a suitable adaptation of the theory of normal surfaces to orbifolds, together with the idea of cutting along spherical 2-orbifolds in increasing order of complication.

2004: complexity of groups Upper estimates on the complexity of a given 3-manifold are experimentally very easy to find, and they typically lead to the exact value. On the other hand, lower estimates are much harder to obtain. Besides those connected to geometry, some bounds can be obtained by considering the fundamental group of the manifold and a suitable notion of complexity for this group. These notions were investigated in the preprint [39] with Ekaterina Pervova, where it was shown that for all Abelian groups, and for all the fundamental groups of the spherical 3-manifolds, the complexity is asymptotically equal (up to linear maps) to the order of the torsion of the Abelianization. As an application, an asymptotically exact estimate of the complexity

was provided for certain infinite families of lens spaces and more general Seifert spaces.

2004-2006: Existence of branched coverings Starting from the Fall of 2004 I started collaborating with Pervova on a very classical problem in the topology of surfaces, known as the *Hurwitz existence problem*. The question is whether there exist a branched covering between closed surfaces matching some preassigned combinatorial requirements. Some necessary conditions, which reduce to the Riemann-Hurwitz formula at least in the orientable case, have been known for a long time, but these conditions were discovered not to be sufficient when the base surface is the sphere, and the knowledge of effective sufficient conditions has been lacking for over a century. Pervova and I have employed three new tools to attack the problem, namely Grothendieck's dessins d'enfants, the idea of factorizing a covering through two non-trivial ones, and a geometric (graph-theoretic) reinterpretation of the existence problem. Using these tools, in the paper [40] accepted for publication in Algebraic and Geometric Topology, we have found several previously unknown cases where the Riemann-Hurwitz conditions are not sufficient, and we have established the sufficiency of conditions having different nature from those appearing in the literature. More such results were obtained in the preprint [43] of November 2006, now submitted for publication, where a completely different approach, based on ideas of Baránski, was exploited. Some of our results were also described in the announcement [41].

2005-2006: Asymptotic complexity estimates for hyperbolic manifolds The value of Matveev's complexity is only known, in the closed case, for a finite number of 3-manifolds, so the issue of computing it for infinite families is a very natural one. In the papers [42, 44] (joint with Matveev and Vesnin) and [45] (joint with Vesnin) I have attacked the question using tools from hyperbolic geometry. The papers contain asymptotic lower and upper bounds for the complexity of three infinite families of hyperbolic manifolds, namely Fibonacci manifolds, Löbell manifolds, and branched covers of two-bridge links. The two bounds differ by fixed (and not too large) factors, so the estimate can be considered to be asymptotically exact up to linear terms.

2006-2007: Collaboration with an anthropologist In the summer of 2006 Maria Luisa Ciminelli, an anthropologist from the University of Venice, wrote and asked me to help her interpret some scientific essays on objects manufactured by Pacific aborigenals from various islands. There handcrafts have shapes of knots, links and surfaces. We corresponded and met several times, and I explained her the mathematics behind these objects. She eventually asked me to contribute to a collective volume she was editing with an article introducing the topology of knots for non-mathematicians, which I did [47].

2006-2007: Collaboration with engineers At the end of 2006 some colleagues from the Department of Electronic Engineering of the University of Pisa came to me with a question arising in the analysis of radar images. The strictly mathematical content of the

matter turned out to be comparatively modest, but I was pleased to be able to help them completely solve a problem with practical applications [48].

2007-2008: Enumeration of hyperbolic knotted graphs Hodgson (Melbourne) and I have started in January 2006 the theoretical work underlying a census of "small" hyperbolic 3-orbifolds. As we were discussing, we understood that the notion of orbifold complexity I have introduced is actually not perfectly suited to the task, and we have worked out a variation which applies to 3-valent (knotted) graphs in 3-manifolds, without any decoration of the edges. After completing this theoretical work, in collaboration with our students Heard and Martelli, we have implemented the census, starting with the case of graphs. We have completely classified the hyperbolic ones (in arbitrary closed 3-manifolds, with hyperbolicity meant in the sense of parabolic edge meridians with thrice punctured geodesic boundary spheres at the vertices) up to complexity 5 (with the restriction that the graph should have no knot component when the complexity is exactly 5), getting many very interesting objects, that we have been able to completely describe together with their many geometric and algebraic invariants coming from the hyperbolic structure. Our results, obtained in collaboration with our students Heard and Martelli, are contained in [46].

2007-2008: Complexity of links Using my results on spherical splitting and complexity of 3-orbifolds, I have developed with Pervova the corresponding new notion of complexity for knots. We have carefully analyzed to what extent this new invariant is additive under connected sum, both along and away from the knot. We have then established upper and lower bounds of complexity in terms of the crossing number, providing evidence for their sharpness. Furthermore we have proposed conjecturally exact upper bounds for the complexity of any torus knot in a lens space and other families of links. Our results are contained in [54]

2007-2008: Manifolds obtained by gluing the faces of the octahedron At the beginning of his 1978 seminal notes on the topology and geometry of 3-manifolds, to anticipate the richness of the topic, Thurston mentions that a vast number of such manifolds can already be obtained by gluing together in pairs the faces of an octahedron. The exact number of such manifolds was however never computed exactly, and in a joint work with Heard and Pervova a filled this gap for the case of orientable manifolds, giving also an extremely detailed geometric description of the manifolds found. To carry out our enumeration and analysis we extensively used computers, together of course with theoretical arguments. The viewpoint we adopted was that of capping off with balls the links of vertices that were spherical, and keeping the non-spherical links as boundary components. For hyperbolic manifolds, that turned out to be a definite majority, the toric components were then regarded as cusps, and the non-toric ones as parts of a geodesic boundary. Our results were published in [50] and announced in [49].

2007-2008: Surface branched covers and geometric 2-orbifolds After having carried out with Pervova extensive research on Hurwitz problem concerning the realizability of candidate surface branched covers by means of various classical techniques (notably permutations, dessins d'enfant, factorization and checkerboard graphs), with my former student Pascali I exploited a new approach using the geometry of 2-orbifolds. Concentrating on the cases of spherical and Euclidean geometry we were able to completely determine the realizability and exceptionality of several infinite families of candidates. In particular, we provided a strong supporting evidence for the conjecture that candidate covers are always realizable if their degree is a prime number, by exhibiting three infinite families of candidates that are asymptotically exceptional with density 1, but are always realizable in prime degree. Our results were published in [52], and some of them were also announced in [51], together with a survey of various techniques employed over the time to face the Hurwitz existence problem.

2007-2008: Colored Turaev-Viro invariants After a rather long period away from quantum topology, I resumed this line of my research by exploiting some of the techniques developed during the study of knot complexity, to analyze a specialization of the Turaev-Viro invariants for knots, obtained by preselecting the color each knot component should have. These invariants had already already been proposed by other authors, but we were able to show their independence from all polynomial invariants coming from the topological quantum theories. We also carefully analyzed their behaviour under connected sum. These results were published in [53].

2007-2008: Number-theoretic interpretations of surface branched covers The work with Pascali naturally prompted connections of the Hurwitz existence problem with questions coming from number theory and algebraic geometry. In a short note [55] with Corvaja and Zannier I then provided alternative interpretations of the results in [52] in terms of automorphisms of elliptic curves, thus also obtaining a new (even if very elaborate) proof of the Fermat-Euler theorem stating that a prime number congruent to 1 modulo 4 can always be expressed as the sum of two squares.

2008-2009: Instances of the Hurwitz problem with hyperbolic 1-dimensional associated Teichmüller space After our applications of the spherical and Euclidean geometry of 2-orbifolds to the problem of realizability of candidate surface branched covers, Pascali and I turned to the hyperbolic case and fully analyzed the instance of candidate covers of triangular orbifolds by orbifolds having 1-dimensional deformation space. We found many new realizable and exceptional branching data, as described in [58].

2009-2011: Dehn surgery on the 5-component chain-link Within their investigations on the virtual Haken conjecture, Dunfield and Thurston remarked that almost

all known closed hyperbolic manifolds are obtained by Dehn surgery along the minimally twisted 5-component chain-link. This link is therefore crucial to hyperbolic geometry (or, at least, for the portion we now know). In a joint work with my student Roukema and my former student Martelli we fully investigated all exceptional Dehn fillings of this link, using techniques similar to those employed by Martelli and me in [30] for the minimally twisted 3-chain link. Our results appear in [60]. One remarkable aspect of them is that they confirm the prediction we had made that the description of the exceptional sets of slopes is actually easier than that given in [30], even if the link is more complicated, because it has a much higher number of symmetries.

2010-2011: Additivity of knot connected sum Using Matveev's theory of roots in a joint paper [59] with Sasso and Pervova I have completely described the instances under which knot complexity is (or is not) additive under connected sum.

2010-2012: Peripheral volume of hyperbolic 3-manifolds With Tocchet [61] I have shown the elementary but surprising fact that for a hyperbolic manifold with both a cusp and a component of geodesic boundary, to maximize the volume encompassed by the union of a cusp neighbourhood and boundary collar one must first individually maximize one of the two. For the case of 3 peripheral components (of any cusp/boundary type) we have established a similar more elaborate result, stating that the maximal volume is attained at some combinatorially extremal configuration.

2012: Spines of vector fields on 3-manifolds with generic tangency to the boundary Using the structure of branched surface on special spines of a 3-manifold M with boundary one can combinatorially describe the vector fields on M with concave tangency to ∂M. In [63] I have shown that using a structure similar to a branching on a more general type of spine of M one can actually recover a completely arbitrary vector field on M, with the only restriction that the behaviour at ∂M should be of generic (stable) type. My results include a description of the sliding moves for generalized spines that translate in combinatorial terms the relation of homotopy relative to the boundary for vector fields.

2012-2013: Spin structures on 3-manifolds via arbitrary triangulations With Benedetti I have introduced the notion of weak branching on a triangulation of a 3-manifold M, and shown [62] that with the addition of some homological weight a weakly branched triangulation carries a spin structure on M. We have then described the combinatorial moves that relate to each other any two weighted weakly branched triangulations defining the same spin structure on M.

2014-2015: Monotonic simplification of link diagrams With Zanellati I have devised certain extensions of the Reidemeister moves on link diagrams, and another move that detects whether a diagram represents a connected sum, that practically perform very well in the task of reducing a diagram to a minimal crossing number status. In fact, we have implemented an algorithm based on our moves, and successfully tested it on a vast collection of known hard diagrams of links, particularly of the unknot, including many provided to us by the mathematical community after we posted the first version of our preprint on the web. Our paper was published as [64].

2016: Exceptionality of an infinite family of branch data Michael Zieve and Umberto Zannier were able to prove that there exist no ramified cover from the torus to the sphere with branch data $(3,\dots,3)$, $(3,\dots,3)$, $(4,2,3,\dots,3)$ using non-trivial algebraic-geometric tools, and they asked me whether I could produce an elementary proof. I analyzed the problem with Tom Ferragut, coming out with an argument that only uses the technique of dessins d'enfant and the homology of the torus (tools that were already available to Riemann and Hurwitz). Our proof is published in [65].

2017-2022: Computation of Hurwitz numbers During these years my scientific activity slowed down due to my administrative duties as head of the teaching programmes in mathematics, chair of the department of mathematics, and vice rector of the university. In any case, I continued working on the existence of surface branched covers realizing specific branch data, concentrating on the harder problem of computing the number of realizations. I have obtained several results for infinite sequences of covers, see [66, 68, 70]. I have also written a paper [69] with a student elucidating the different ways the number of realizations can be defined, and a survey paper [71].

2023: Solution of the Hurwitz problem with a length-2 partition In [73] with a former student of mine I have completed Pakovich's program to investigate the realizability of candidate surface branched covers with a length-2 partition, settling the case where the candidate covering surface is not the sphere.

2023: Apparent contours of properly immersed curves in surfaces with boundary In [74] I have introduced a combinatorial encoding for properly immersed curves in compact surfaces with non-empty boundary. On such a surface I consider a generic traversing flow, namely the set of the orbits of a non-singular vector field, with all the orbits being points of intervals, subject to a natural genericity condition (many such flows always exist). I show that this flow can be encoded by a *flow-spine*, that is a uni-trivalent graph with singular smooth structure at the trivalent vertices. Then for a properly immersed curve I take the *apparent contour* on the flow-spine, given by a collection of labelled extra bivalent vertices and weights on the edges of the graph. I show that the apparent contour fully determines the curve up to diffeomorphism, and then I translate

into combinatorial moves both the homotopy of the curve through properly immersed curves within a fixed traversing flows, and the simultaneous homotopy of the curve and the flow.

2025: Link and band diagrams on spines In [75] I have founded a theory of diagrams of links and bands on 3-manifold almost special spines and flow-spines. This extends the classical theory of Reidemeister moves in several very general contexts.

10 Scientific activity: work in progress and projects

Apparent contours of surfaces in 3-manifolds I am currently working on extending the theory developed in [74] to the case of 1 dimension up. This will use the notion defined in [63] of flow-spine for a generic traversing flow on a 3-manifold. The idea is again that of encoding a surface by means of its apparent contour on a flow-spine, and to introduce combinatorial moves to translate homotopy. This will probably be confined to closed embedded surfaces, but one can be optimistic and hope to be able to do again the properly immersed case. The theory of apparent contours of surfaces for the basic case of the constant vertical flow in 3-space, where the flow-spine is just the horizontal plane, is thoroughly described in Bellettini, Giovanni; Beorchia, Valentina; Paolini, Maurizio; Pasquarelli, Franco. "Shape reconstruction from apparent contours. Theory and algorithms." Computational Imaging and Vision, Vol. 44. Springer, Heidelberg, 2015.

Invariants for curves The theory of planar curves has a long history, with connections to deep areas of geometric topology, such as for instance contact structures. Two foundational papers in this area are: Arnold, Vladimir I. *Plane curves, their invariants, perestroikas and classifications.* With an appendix by F. Aicardi. In: "Singularities and Bifurcations," Advanas Soviet Mathematics, Vol. 21 (American Mathematical Society, Providence, RI, 1994), pp. 33–91. And Fenn, Roger; Taylor, Paul. *Introducing doodles.* In: "Topology of low-dimensional manifolds" (Proc. Second Sussex Conf., Chelwood Gate, 1977), 37–43, Lecture Notes in Math., 722, Springer, Berlin, 1979. Since my theory of apparent contours of curves on flow-spines gives a way to study curves in more general surfaces than the plane, I am planning to study the mathematics that grew from the mentioned seminal papers and extend some of it to the more general context.

More instances of the Hurwitz existence problem In the last several years I have given various contributions to the understanding of which candidate combinatorial surface branched covers can be realized by actual maps. However many cases still remain mysterious, and I am planning to keep working on unveiling some of them. I believe in particular that some techniques developed in recent years could be pushed forward to cover more cases of existence and/or exceptionality. I refer in particular to the geometric arguments given in Xuwen Zhu, *Spherical conic metrics and realizability of branched covers*, Proc.

Amer. Math. Soc. **147** (2019), 1805-1815, and to the algebraic combinatorial ones in Jijian Song, Bin Xu, *On rational functions with more than three branch points*, Algebra Colloq. 27 (2020), 231–246. So I will study these and related papers deeply and work on extending their achievements.

Computation of link invariants and skein modules in 3-manifolds other than the sphere After their original definition through complicated algebraic machineries, the quantum invariants of links were rediscovered in purely combinatorial terms through planar diagrams. In this context the Kauffman bracket lead to the definition of the skein module of a 3-manifold other than the sphere. I plan to exploit the machinery developed in [75] to extend the diagrammatic approach to link invariants and in particular to compute she skein module of some 3-manifolds for which it is still unknown.

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